



বিদ্যাসাগর বিশ্ববিদ্যালয়  
VIDYASAGAR UNIVERSITY  
Question Paper

**B.Sc. General Examination 2022**

(Under CBCS Pattern)

**Semester - VI**

**Subject : MATHEMATICS**

**Paper : SEC 4 - T**

**Full Marks : 40**

**Time : 2 Hours**

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

[ PROBABILITY AND STATISTICS ]

1. Answer any **four** questions : 5×4=20
- (a) A point  $R$  is chosen at random on a line segment  $OA$  of length  $2a$ . Find the probability that the area of the rectangle with its sides  $OR$  and  $RA$  will never exceed  $\frac{a^2}{2}$ . 5
- (b) (i) If  $X$  is a Poisson ( $\mu$ ) variate, then find the probability distribution of  $X^3$ .  
(ii) Find the mean and variance of the rectangular distribution. 3+2
- (c) (i) Show that correlation coefficient between two random variables is numerically 1, if and only if they are linear.

(ii) The regression lines of two random variables  $X$  and  $Y$  are  $2x + 3y = 5$ ,  $5x + 2y = 8$ . Find their correlation coefficient. 2+3

(d) For a normal  $(m, \sigma)$  distribution, prove that  $\mu_{2r} = 1.3.5 \dots (2r-1)\sigma^{2r}$ , where  $\mu_r$  represents  $r$ -th order central moments and  $r$  being a positive integer. 5

(e) If  $X$  is a binomial  $(n, p)$  variate, then prove that  $\mu_{k+1} = p(p-1) \left( nk\mu_{k-1} + \frac{d\mu_k}{dp} \right)$ , where  $\mu_k$  is the  $k^{\text{th}}$  order central moment. 5

(f) Two discrete random variables  $X$  and  $Y$  have joint probability mass function given by the following table :

		Y		
		1	2	3
X	1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
	2	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
	3	$\frac{1}{12}$	$\frac{1}{12}$	0

Compute the probability of each of the following events :

(i)  $X \leq 1.5$  (ii)  $XY$  even ((iii)  $Y$  is even given that  $X$  is even. 5

2. Answer any **two** questions : 10×2=20

(a) (i) State and prove Baye's theorem.

(ii) Show that the probability that exactly one of the events A and B occurs is  $P(A) + P(B) - 2P(AB)$ .

(iii) If  $f(x) = A \text{Exp} \left[ -\frac{1}{2b^2}(x-a)^2 \right]$ ,  $-\infty < x < \infty$ , find  $A$  so that  $f(x)$  is a probability density function. 4+3+3

- (b) If the joint pdf of the random variables  $X, Y$  is

$$f_{x,y}(x,y) = \begin{cases} kx(3x+y), & 0 \leq x \leq 3, 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}.$$

Find (i)  $P(X+Y < 2)$  (ii) the marginal distributions of  $X$  and  $Y$ , Investigate whether  $X$  and  $Y$  are independent. 5+4+1

- (c) (i) If  $X$  is uniformly distributed over  $[1, 2]$ , find  $K$  so that  $P(X > K + E(X)) = \frac{1}{6}$ .

- (ii) A jar contains three white and three red balls. The balls are drawn at random from the jar and placed on a table in the order drawn. What is the probability that balls are drawn in the order white, red, red, white, red, white? 5+5

- (d) (i) A dart is thrown at random on a square target board having vertices  $(1, 0)$ ,  $(0, 1)$ ,  $(-1, 0)$  and  $(0, -1)$ , the point at which the dart hits the board being  $(X, Y)$ , find the marginal density functions of  $X$  and  $Y$  and show that they are dependent.

- (ii) Let  $X_1, X_2, X_3$  be three independent  $N(\mu, 1)$  variables. Prove that

$$P[X_1 - 2X_2 + X_3 > 0] = \frac{1}{2}. \quad \text{6+4}$$

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**OR****[ FORECASTING ]**

1. Answer any *four* questions : 5×4=20

- (a) Discuss the overall purpose people have for investing. Define investment.
- (b) Briefly discuss the five fundamental factors that influence the risk premium of an investment.
- (c) On August 15, you purchased 100 shares of stock in a Company X at \$65 a share and a year later you sold it for \$61 a share. During the year, you received dividends of \$3 a share. Compute your HPR and HPY on your investment in X.
- (d) Discuss the process of double exponential smoothing.
- (e) Discuss the key factors of time series in business forecasting.
- (f) What are the disadvantages of forecasting?

2. Answer any *two* questions : 10×2=20

- (a) Given the weekly demand data, what are the exponential smoothing forecasts for Periods 2-10 using :

A)  $\alpha = 0.10$ , B)  $\alpha = 0.60$

Assume  $F_1 = D_1$

Week	1	2	3	4	5	6	7	8	9	10
Demand	820	775	680	655	750	802	798	689	775	?

- (b) Given forecast errors of -1, 4, 8 and -3, what is the mean absolute deviation?
- (c) In a simple linear regression model,  $y_i = \beta x_i + \varepsilon_i$  where  $\varepsilon_i$  is  $N(0, \sigma^2)$ ,  $i = 1, 2, \dots, n$ . Determine an unbiased estimator of  $\sigma^2$ .
- (d) Suppose there are 5 explanatory variables (including an intercept term) in the model  $y = X\beta + \varepsilon$  and  $n = 26$ . Further, the sum of squares due to total is obtained as 300 and sum of squares due to error is obtained as 100. Find the value of  $F$ -statistic in the context of analysis of variance.

**OR****[ PORTFOLIO OPTIMIZATION ]**

1. Answer any *four* questions : 5×4=20

- (a) Define (i) Beta of a portfolio (ii) Security market line.
- (b) You have a portfolio with a beta of 0.84. What will be the new portfolio beta if you keep 85% of your money in the old portfolio and 14% in a stock with a beta of 1.93?
- (c) What are some of the benefits of diversification?
- (d) Use the information in the following to answer the questions below :

State of Economy	Probability of State	Return on A in State	Return on B in State
Boom	35%	0.040	0.210
Normal	50%	0.030	0.080
Recession	15%	0.046	-0.010

- (i) What is the expected return of each asset?
- (ii) What is the variance of each asset?
- (e) Define the Sharpe, Treynor, and Jensen measures of portfolio performance evaluation.
- (f) Explain different types of risks in a portfolio optimization.

2. Answer any *two* questions : 10×2=20

- (a) Explain Sharpe model for determining the efficient portfolio set.
- (b) Explain the capital asset pricing model (CAPM).
- (c) Explain Markowitz approach to portfolio selection.
- (d) Write a note on financial markets.

