



বিদ্যাসাগর বিশ্ববিদ্যালয়  
VIDYASAGAR UNIVERSITY

Question Paper

**B.Sc. Honours Examinations 2020**

(Under CBCS Pattern)

**Semester - I**

**Subject: MATHEMATICS**

**Paper: GE 1-T**

**Full Marks : 60**

**Time : 3 Hours**

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

Answer any **three** from the following questions :

3×20

1. (a) Show that the curve  $y^3 = 8x^2$  is Concave to the foot of the ordinate everywhere except at Origin. 4
- (b) State some natures of Hyperbolic Sine. 4
- (c) If  $y = 2 \cos x (\sin x - \cos x)$ , show that  $y_{10}(0) = 2^{10}$ . 6
- (d) Find the envelopes of the straight line  $\frac{x}{a} + \frac{y}{b} = 1$  where the parameters  $a$  and  $b$  are connected by the relation  $a^2 + b^2 = c^2$  6

2. (a) If  $y = (ax + b)^m$  find  $D^n(ax + b)^m$ . 4
- (b) Evaluate  $\lim_{x \rightarrow 0} (\cos mx)^{\frac{n}{x^2}}$ . 4
- (c) Find the length of a quadrant of the circle  $r = 2a \sin \theta$ . 4
- (d) Evaluate  $\int_0^{\pi/2} \sin^8 x \cos^6 x dx$ . 4
- (e) The circle  $x^2 + y^2 = a^2$  revolves about the  $x$ -axis. Show that the surface area and the volume of the sphere thus generated are respectively  $4\pi a^2$  and  $\frac{4}{3}\pi a^3$ . 4
3. (a) Evaluate  $\int_0^{\pi/4} \tan^5 x dx$ . 4
- (b) Find the volume of the solid generated by revolving the part of parabola  $x^2 = 4ay, a > 0$  between the ordinates  $y = 0$  and  $y = a$  about its axis. 4
- (c) Find the area of the smaller portion enclosed by the curves  $x^2 + y^2 = 9$  and  $y^2 = 8x$ . 6
- (d) Trace out the curve cycloid  
 $x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$  6
4. (a) Through what angle must be the axis be turned to remove  $xy$  term from  $7x^2 + 4xy + 3y^2 = 0$ . 4
- (b) If pair of lines  $x^2 - 2pxy - y^2 = 0$  and  $x^2 - 2qxy - y^2 = 0$  be such that each pair bisects the angles between the other pair, prove that  $pq + 1 = 0$ . 4
- (c) Find the equation of the cylinder whose generators are parallel to the straight line  $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$  and whose guiding curve is  $x^2 + y^2 = 9, z = 1$ . 6

- (d) The plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  meets the co-ordinate axes  $A, B, C$ . Find the equation of the cone generated by the straight lines drawn from  $O$  to meet the circle  $ABC$ . 6
5. (a) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord. 4
- (b) Find the equation of the circle on the sphere  $x^2 + y^2 + z^2 = 49$  whose centre is at the point  $(2, -1, 3)$ . 4
- (c) Show that the straight line  $r \cos(\theta - \alpha) = p$  touches the conic  $\frac{l}{r} = 1 + e \cos \theta$  if  $(l \cos \alpha - ep)^2 + l^2 \sin^2 \alpha = p^2$ . 6
- (d) Find the equation of the plane which passes through the point  $(2, 1, -1)$  and is orthogonal to each of the planes  $x - y + z = 1$  and  $3x + 4y - 2z = 0$ . 6
6. (a) Find the differential equation of all circles passing through the origin having centres on the  $x$ -axis. 4
- (b) Find an integrating factor of the differential equation  $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$  4
- (c) Find the general and the singular solutions of  $y = px + \sqrt{a^2p^2 + b^2}$ . 6
- (d) Reduce the differential equation  $(px^2 + y^2)(px + y) = (p+1)^2$  to Clairaut's form by the substitution  $u = xy, v = x + y$  and then find the general solution. Where  $p = \frac{dy}{dx}$ . 6
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