বিদ্যাসাগর বিশ্ববিদ্যালয়
VIDYASAGAR UNIVERSITY
Question Paper

B.Sc. Honours Examinations 2022
(Under CBCS Pattern)
Semester - II
Subject : MATHEMATICS
Paper : C 3 - T
[REAL ANALYSIS]

Full Marks : 60
Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any *five* questions : 2×5=10

   (a) Define least upper bound of a bounded set and obtain it for the set

   $$A = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \ldots \frac{n}{n+1}, \ldots \ldots \right\}$$

   (b) Define point of accumulation of a set and find all the points of accumulation of

   the set $E = \left\{ \frac{1}{m} + \frac{1}{n} / m, n = 1, 2, 3, \ldots \ldots \right\}$.

   (c) Prove that if A and B are two closed sets, then $A \cup B$ and $A \cap B$ are both closed sets.

(d) Show that $\left\{\dfrac{3n+1}{n+2}\right\}$ is a bounded sequence.

(e) Prove that every convergent sequence is bounded. Is the converse true? Justify.

(f) Prove that the Series $\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{n(n+1)}$ converges.

(g) If $\displaystyle\sum_{n=1}^{\infty}a_n$ is a convergent series, then prove that $\displaystyle\lim_{n\to\infty}a_n=0$.

(h) Define compact set with an example.

2. Answer any *four* questions : 5×4=20

(a) Define countability of a set. Show that the set of all real numbers is not countable.

(b) State and prove Archimedean property of real numbers.

(c) If a set S is open, then prove that its complement is a closed set. Is the converse true? Justify.

(d) Define Cauchy Sequence. Prove that the sequence $\left\{n^2\right\}$ is not a Cauchy Sequence.

(e) Prove that every bounded sequence has a convergent subsequence.

(f) Prove that $1+\dfrac{1}{2!}+\dfrac{1}{4!}+\dfrac{1}{6!}+\dots\dots$ converges.

3. Answer any *three* questions : 10×3=30

(a) What do you mean by convergence, absolute convergence and conditional convergence of a series of real numbers? Prove that absolutely convergence imply convergence. Classify as to divergent, conditionally convergent or absolutely of the following series :

(i) $1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\dots\dots$

(ii) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

(iii) $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + ...$ 3+1+6

(b) If a sequence $\{x_n\}$ of real numbers is monotonic increasing and bounded above, then prove that it converges to its exact upper bound. Prove that the sequence $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}$ is monotonic increasing and bounded above.   5+5

(c) (i) State and prove Bolzano-Weierstrass theorem for sequences.

(ii) Using Cauchy's general principle of convergence prove that $\{x_n\}$, where $x_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + (-1)^{n-1} \cdot \frac{1}{n}$, is a convergent sequence.   5+5

(d) (i) State and prove Heine-Borel theorem. Give an illustration which justify Heine-Borel theorem.

(ii) State and prove density property of real numbers.   4+3+3

(e) (i) Examine if the following series converge :

(i) $\sum\limits_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$ (ii) $\sum\limits_{n=1}^{\infty} \frac{n+1}{n+2}$ (iii) $\sum\limits_{n=1}^{\infty} \log\left(1 + \frac{1}{n}\right)$

(ii) Given $0 < x_1 < x_2$. If each $x_n = \frac{x_{n-1} + x_{n-2}}{2}$,

then prove that $\{x_n\} \to \frac{1}{3}(x_1 + 2x_2)$ as $n \to \infty$.   6+4