বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2022

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : DSC 1B/2B/3B - T

[DIFFERENTIAL EQUATIONS]

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***ten*** questions : 2×10=20

(a) Determine the degree and order of the differential equation

$$x^3 \frac{d^2 y}{dx^2} + \cos x \left(\frac{dy}{dx}\right)^2 + (\sin x)\, y = 0 .$$

(b) State the existence and uniqueness theorem of ODE.

(c) Show that the equation $\left(x^3 - 3x^2 y + 2xy^2\right) dx - \left(x^3 - 2x^2 y + y^3\right) dy = 0$ is exact and find the solution if $y = 1$ when $x = 1$.

(d) Find the integrating factor of $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^2$.

(e) Solve : $\left(2xy+e^{x}\right)y\,dx-e^{x}dy=0$.

(f) If $y_1\left(x\right)=e^{-3x}$ and $y_2\left(x\right)=e^{2x}$ are two solutions of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$. Show that $y_1\left(x\right)$ and $y_2\left(x\right)$ are linearly independent.

(g) Find the value of $\frac{1}{\left(D+2\right)}e^{-2x}\sin 3x$.

(h) Prove that e^{x^2} is an integrating factor of $\left(x^2+xy^4\right)dx+2y^3dy=0$ and hence solve it.

(i) Obtain the complete primitives and singular solution of the clairaut's form $y=Px+P-P^2$.

(j) Define singular solution of an ODE.

(k) Solve : $\frac{xdx}{y^2z}=\frac{dy}{xz}=\frac{dz}{y^2}$, z is a function of x and y.

(l) Eliminate the arbitrary constant a from the given relation $z=a\left(x+y\right)$.

(m) Solve : $\frac{dy}{dx}=\left(y+3x\right)^2$.

(n) If $\frac{du\left(x\right)}{dx}=v\left(x\right),\frac{dv\left(x\right)}{dx}=u\left(x\right)$ and $u(0) = 1$ and $v(0) = 1$. Find $u(x)$.

(o) Solve : $x^2p+yq=z^2$.

2. Answer any ***four*** questions : 5×4=20

(a) Solve : $\left(x^3+xy^4\right)dx+2y^3dy=0$.

(b) Find the integral surface of the linear partial differential equation $\left(x-y\right)p+\left(y-x-z\right)q=z$ through the circle $x^2+y^2=1,\ z=1$.

(c) Solve the differential equation $\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4y=e^{2x}+e^x+3e^{-x}$.

(d) Solve by the method of variation of parameter $\frac{d^2y}{dx^2}+a^2y=\sec(ax)$.

(e) Solve the simultaneous equations :

$(D-17)x+(2D-8)y=0$
$(13D-53)x-2y=0$

(f) Solve : $x^2dy+y(x+y)dx=0$.

3. Answer any ***two*** questions : 10×2=20

(a) (i) Solve : $y(2xy+e^x)dx-e^xdy=0$.

(ii) Solve and find the singular solution of $x^3p^2+x^2py+a^3=0$.

(b) (i) Solve : $(D^2-6D+25)y=2e^{3x}\cos 4x+8e^{3x}(1-2x\sin 4x)$.

(ii) Solve by the method of variation of parameters $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}=1,\ x>0$ is being given by $y=x^{-1},\ y=1$ and $y=x$ are three linearly independent solutions of its reduced equation.

(c) (i) Solve : $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$.

(ii) Find the partial differential equation arising from $\phi\left(\frac{z}{x^3},\frac{y}{x}\right)=0$, where ϕ is an arbitrary function of its argument.

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