B.Sc. First Semester Examination(ESE) 2024

(CCFUP : NEP)

(3-Year UG- Programme)

MATHEMATICS

Paper Code : UG/I/MATH/3/SE-1P

Paper Name: MATLAB-1

Full Marks - 40 (Viva-5, Note book- 5, Practical 30)

Time - 3 hours

The figures in the margin indicate full Marks. Candidates are required to give their own words as far as practicable. Illustrate the answers wherever necessary. The questions are to be selected by lottery.

Group - A [10 Marks]

Answer any one question :

1. Write a program in MATLAB to find the sum and product of a list of numbers in an array without library function.
2. Write a program in MTLAB to find the maximum and minimum of a list of numbers in an without library function.
3. Write a program in MTLAB to find the sum and product of a list of numbers in a sub-array without library function.
4. Write a program in MATLAB to find the maximum and minimum of a list of numbers in a sub-array without library function.
5. Write a program in MATLAB to find a sub-matrix of the given matrix.
6. Write a program in MATLAB to find the column sum and product of the given matrix without library function.
7. Write a program in MATLAB to find the column max and min of the given matrix.
8. Write a program in MATLAB to find the row sum and product of the given matrix without library function.

9. Write a program in MATLAB to find the row max and min of the given matrix without library function.

Group - B [10 Marks]

Answer any one question :

1. Write a program in MATLAB to plot of the curves e^{ax+b} and log $(ax+b)$.
2. Write a program in MATLAB to plot of the curves sin $(ax+b)$ and cos $(ax+b)$.
3. Write a program in MATLAB to plot of the curves $|ax+b|$ and $-|ax+b|$.
4. Write a program in MATLAB to plot the graphs of a polynomial of degree 4, its derived graph and the 2^{nd} derivative of the graph.
5. Write a program in MATLAB to plot the graphs of a polynomial of degree 5, its derived graph and the 2^{nd} derivative of the graph.

Group - C [10 Marks]

Answer any one question :

1. Write a program in MATLAB to sketch the trochoid
   $x = a\theta - b\sin\theta, \quad y = a - b\cos\theta.$
2. Write a program in MATLAB to sketch the cycloid
   $x = a(\theta - \sin\theta), \; y = a(1-\cos\theta).$
3. Write a program in MATLAB to sketch the epicycloids
   $x = (a+b)\cos\theta - b\cos\left(\frac{a+b}{b}\theta\right), \; y = (a+b)\sin\theta - b\sin\left(\frac{a+b}{b}\theta\right)$
4. Write a program in MATLAB to sketch the hypocycloid

   $x = (a-b)\cos\theta + b\cos\left(\frac{a-b}{b}\theta\right), \; y = (a-b)\sin\theta - b\sin\left(\frac{a-b}{b}\theta\right)$
5. Write a program in MATLAB for tracing the conic
   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$
6. Write a program in MATLAB for tracing the conic
   $\frac{l}{r} = 1 - e\cos\theta.$
7. Write a program in MATLAB for sketching the ellipsoid
   $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$
8. Write a program in MATLAB for sketching the hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$
9. Write a program in MATLAB for sketching the hyperboloid of one sheet $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$
10. Write a program in MATLAB for sketching the elliptic cone
    $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$
11. Write a program in MATLAB for sketching the elliptic paraboloid
    $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$
12. Write a program in MATLAB for sketching the hyperbolic paraboloid $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

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B.Sc. First Semester Examination(ESE) 2024

(CCFUP : NEP)

(3-Year UG- Programme)

MATHEMATICS

Paper Code : UG/I/MATH/3/MI-C1T

Paper Name : Calculus, Geometry & Ordinery Differentiel Equations

Full Marks - 60 **Time - 3 hours**

The figures in the margin indicate full Marks. Candidates are required to give their own words as far as practicable. Illustrate the answers wherever necessary.

Unit-I (Differential Calculus)

[16 Marks]

1. **Answer any *three* from the followings :** **3×2=6**

   a) Show that the point of inflexion of the curve $y^2=(x-a)^2(x-b)$ lies on the line $3x + a = b$.

   b) Evaluate : $\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

   c) Find the asymptotes of $y = xe^{\frac{1}{x}}$

   d) Find the envelope of the line $x\cos\alpha + y\sin\alpha = l\sin\alpha\cos\alpha$

   e) Find y_5 where $y = x^3 \log x$

2. **Answer any *one* from the followings :** **1×10=10**

   a) i) If ρ_1 and ρ_2 be the radii of curvature at the ends of the conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, prove that

   $(\rho_1^{2/3} + \rho_2^{2/3})(ab)^{2/3} = a^2 + b^2$

   ii) Find the asymptotes of the curve

$$x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$$

b) i) Find the envelope of the family of parabolas $\left(\frac{x}{a}\right)^{\frac{1}{2}}+\left(\frac{y}{b}\right)^{\frac{1}{2}}=1$, where $a+b=c$, a, b being the parameter

ii) Prove that the radius of curvature of the curve $y=c\cosh\left(\frac{x}{c}\right) is \frac{y^2}{c}$.

Unit-II (Integral Calculus)

[14 marks]

3. Answer any *two* from the followings : 2×2=4

a) Find the area in the first quadrant by $x=0, y=0$ and $\sqrt{x}+\sqrt{y}=\sqrt{a}$.

b) If $I_n=\int_0^{\pi/2} x^n \sin x dx$, ($n$>1) prove that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$

c) Show that the arc of the upper half of the cardioids $r=a(1-\cos\theta)$ is bisected at $\theta=\frac{2\pi}{3}$.

d) Find the volume generated by revolving about OX the area bounded by the loop of the curve $y^2(a+x)=x^2(a-x)$.

4. Answer any *two* from the followings : 2×5=10

a) The area enclosed between the arcs of the parabola $y^2=4ax$ from the vertex to one extremities of the latus rectum is revolved about the corresponding chord. Find the volume of the spindle thus generated.

b) If m, n are positive integers then show that

9. Answer any *one* from the followings : 1×5=5

a)Solve: $(2xy^4e^y+2xy^3+y)dx+(x^2y^4e^y-x^2y^2-3x)dy=0$.

b) Solve: $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$.

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$x = \alpha y + \beta = \gamma z + \delta$ are co-planner if $(\gamma - c)(a\beta - b\alpha) = (\alpha - a)(c\delta - d\gamma)$.

ii) Reduce the equation $4x^2 + 4y^2 + 4z^2 - 2x - 14y - 22z + 33 = 0$ to its cannonial form and determine the type of the quadric represented by it.

b) i) Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lies on the intersection of the ellipsoid and the cone $\frac{a^2\alpha(b^2 - c^2)}{x} + \frac{b^2\beta(c^2 - a^2)}{y} + \frac{c^2\gamma(a^2 - b^2)}{z} = 0$.

ii) Find the equation of the generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ though a point of principal elliptic section. Hence show that the projections of the generators of a hyperboloid on co-ordinate planes are tangents to the section of the hyperboloid by that plane.

Unit-IV (Differential Equation)

[9 Marks]

8. **Answer any *two* from the followings : 2×2=4**

a) State the necessary and sufficient condition for which the differential equation $f(x, y)\frac{dy}{dx} + g(x, y) = 0$ is exact.

b) Find the integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$.

c) Obtain the general and singular solution of $y = px + \frac{a}{p}$.

d) Verify whether x^3 is an I.F. of $2\sin y^2 dx + xy\cos y^2 dy = 0$ or not.

$$I_{m,n} = \int_0^1 x^m (1-x)^n \, dx = \frac{m!n!}{(m+n+1)!}.$$

c) Find the length of the loop of $3ay^2 = x(x - a)^2$.

d) Show that the area bounded by one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and the x-axis is $3\pi a^2$.

Unit-III (Geomatry)

[21 Marks]

5. **Answer any *three* from the followings : 3×2=6**

a) Find the equation of the curve $x^2 + y^2 = a^2$ when the axes rotate by angle 60º.

b) Find the nature of the conic $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$.

c) Find the polar equation of the straight line joining the two points $\left(1, \frac{\pi}{2}\right)$ and $(2, \pi)$.

d) Find the distance of the point (–1, 1, –2) from the plane passing through the points (1, –1, 1), (–2, 1, 3), (4, –5, –2).

e) Find the equation of the cone whose vertex is the origin and base is the circle $x = a$, $y^2 + z^2 = b^2$.

6. **Answer any *one* from the followings : 1×5=5**

a) Prove that the centres of spheres which touch the straight line $y = mx$, $z = c$ and $y = -mx$, $z = -c$ lie on the surface $mxy + cz(1 + m^2) = 0$.

b) Prove that the locus of the middle point of any focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ is $r(1 - e^2\cos^2\theta) = el\cos\theta$.

7. **Answer any *one* from the followings : 1×10=10**

a) i) Prove that the straight lines $x = ay + b = cz + d$ and

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MATHEMATICS

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Paper Name: MATLAB-1

Full Marks - 40 (Viva-5, Note book -5, Practical -30)

Time - 3 hours

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Group - A [10 Marks]

Answer any *one* questions :

1. Write a program in MATLAB to find the sum and product of a list of numbers in an array without library function.
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Group - B [10 Marks]

Answer any *one* question :

1. Write a program in MATLAB to plot of the curves e^{ax+b} and log $(ax+b)$.
2. Write a program in MATLAB to plot of the curves sin $(ax+b)$ and cos $(ax+b)$.
3. Write a program in MATLAB to plot of the curves $|ax+b|$ and $-|ax+b|$.
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5. Write a program in MATLAB to plot the graphs of a polynomial of degree 5, its derived graph and the 2nd derivative of the graph.

Group - C [10 Marks]

Answer any one question :

1. Write a program in MATLAB to sketch the trochoid

   $x = a\theta - b\sin\theta, \quad y = a - b\cos\theta.$

2. Write a program in MATLAB to sketch the cycloid

   $x = a(\theta - \sin\theta), \; y = a(1-\cos\theta).$

3. Write a program in MATLAB to sketch the epicycloids

   $x = (a+b)\cos\theta - b\cos\left(\frac{a+b}{b}\theta\right), \; y = (a+b)\sin\theta - b\sin\left(\frac{a+b}{b}\theta\right)$

4. Write a program in MATLAB to sketch the hypocycloid

   $x = (a-b)\cos\theta + b\cos\left(\frac{a-b}{b}\theta\right), \; y = (a-b)\sin\theta - b\sin\left(\frac{a-b}{b}\theta\right)$

5. Write a program in MATLAB for tracing the conic

   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$

6. Write a program in MATLAB for tracing the conic

   $\frac{l}{r} = 1 - e\cos\theta.$

7. Write a program in MATLAB for sketching the ellipsoid

   $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

8. Write a program in MATLAB for sketching the hyperboloid of one sheet $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$
9. Write a program in MATLAB for sketching the hyperboloid of one sheet $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$
10. Write a program in MATLAB for sketching the elliptic cone

    $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

11. Write a program in MATLAB for sketching the elliptic paraboloid

    $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$

12. Write a program in MATLAB for sketching the hyperbolic paraboloid $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

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ii) Find the equation of the cylinder whose generators are parallel to the line $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1,\ z = 3.$ 5

Unit-IV(Differential Equation)

[9 Marks]

8. Answer any *two* from the followings : 2×2=4

a) Solve: $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$.

b) Solve the differential equation $(x+y)dx + (x-y)dy = 0$ by finding an appropriate integrating factor.

c) Solve the Bernoulli equation $\frac{dy}{dx} + 2y = y^2$.

9. Answer any *one* from the followings : 1×5=5

a) Solve the differential equation: $\frac{dy}{dx} + y\cot x = 2\cos x$.

b) Solve the differential equation:

$(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$

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B.Sc. First Semester Examination(ESE) 2024

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(4-Year UG- Programme)

MATHEMATICS

Paper Code : UG/I/MATH/4/MI-1T

Paper Name : Calculus, Geometry & Ordinery Differentiel Equations

Full Marks - 60 **Time - 3 hours**

The figures in the margin indicate full Marks. Candidates are required to give their own words as far as practicable. Illustrate the answers wherever necessary.

Unit-I(Differential Calculus)

[16 Marks]

1. Answer any *three* from the followings : 3×2=6

a) Prove that the radius of curvature of the circle $x^2 + y^2 = a^2$ is a.

b) Find $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$.

c) Find the nth derivative of $x\log x$.

d) Compute the second derivative of the function $f(x) = ax + b.n\cos x$ and use it to identify intervals where the function is concave up or concave down.

e) Find the envelope of the family of curves $y = e^x\cos(x) + C$ for some constant C.

2. Answer any *one* from the followings : 1×10=10

a) i) If $y = \frac{x^3}{x^2 - 1}$, then prove that $(y_n)_0 = \begin{cases} 0, & \text{if } n \text{ is even;} \\ -n! & \text{if } n \text{ is odd} \end{cases}$ $n > 1$. 5

ii) Find all the asymptotes of the curve 5

$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$.

b) i) Find a, b in order that $\lim_{x \to 0} \frac{a \sin 2x - b \sin x}{x^3} = 1$. 5

ii) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters are connected by $a^2 + b^2 = c^2$ (c being a given constant). 5

Unit-II (Integral Calculus)

[14 Marks]

3. **Answer any *two* from the followings : 2×2=4**

a) Given the parametric equations $x = t^2$ and $y = t^3$, find the arc length of the curve from $t = 0$ to $t = 2$.

b) If $m \neq n$, then find the value of $\int_0^{\pi} \cos mx \sin nx dx$.

c) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta,\ n \in \mathbb{N}$, the prove that $I_{n+1} + I_{n-1} = \frac{1}{n}$

4. **Answer any *two* from the followings : 2×5=10**

a) Find the value of $\int_0^{\frac{\pi}{2}} \sin^n x dx$ by using reduction formula. 5

b) Find the area enclosed by the curve given by the parametric equations $x = \sin t$ and $y = \cos t$ for t in the range $[0, 2\pi]$. 5

c) If $I_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \sin nx dx$, where m, n being positive integers, prove that $I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + ... + \frac{2^m}{m}\right]$. 5

Unit-III (Geometry)

[21 Marks]

5. **Answer any *three* from the followings : 3×2=6**

a) Find the equation of the curve $3x^2 + 3y^2 + 6x - 18y = 14$ referred to parallel axes through the point $(-1, 3)$.

b) Given the polar equation $r = \frac{3}{1 + 2\cos\theta}$, classify the conic it represents.

c) Find the equation of the plane section of a cone with equation $x^2 + y^2 - z^2 = 0$ for the plane $z = 1$.

d) If P be a variable point such that its distance from the xz-plane is always equal to its distance from the y-axis, then show that the locus of P is a cone.

e) Show that the straight line $x - 1 = y - 2 = z + 1$ lies entirely on the surface $z^2 - xy + 2x + y + 2z - 1 = 0$.

6. **Answer any *one* from the followings : 1×5=5**

a) If r_1 and r_2 be two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2 \cos^2 \theta}$, then show that $\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle.

7. **Answer any *one* from the followings : 1×5=5**

a) i) If PSP' and QSQ' be two perpendicular focal chord of a conic with focus S, then prove that $\frac{1}{SP.SP'} + \frac{1}{SQ.SQ'}$ is constant.

ii) Find the equation of the right circular cone whose vertex is the origin and whose axis is $\frac{x}{3} = \frac{y}{1} = \frac{z}{1}$. 5

b) i) Reduce the equation $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to the canonical form and hence find the nature of the conic. 5

ii) Show that the equation of the sphere through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0 = x^2 + y^2 + z^2 + 4x + 5y - 6z + 2$ and having its centre on the plane $4x - 5y - z = 3$ is $x^2 + y^2 + z^2 + 7x + 9y - 11z - 1 = 0$.

b) i) Show that the equation of the right circular cylinder of radius 2 whose axis passes through the point (1, 2, 3) and has direction ratios (2, –3, 6) is $45x^2 + 40y^2 + 13z^2 + 36yz - 24zx + 12xy - 42x - 280y - 126z + 294 = 0$.

ii) Find the equations of the generators of the hyperboloid $x^2 + 4y^2 - 9z^2 = 25$ passing through the point (3, 2, 0).

Unit-IV (Differential Equation)

[9 Marks]

8. Answer any *two* from the followings : 2×2=4

a) Find an integrating factor of the differential equation $x^2 y dx - (x^3 + y^3) dy = 0$

b) Show that the solution of $\frac{dy}{dx} + Py = Q$ can also be written in the form $y = \frac{Q}{P} - e^{-\int P dx}\left[c + \int e^{\int P dx} d\left(\frac{Q}{P}\right)\right]$

c) Solve the differential equation $xy(p^2 - 1) = (x^2 - y^2)p$, where $p = \frac{dy}{dx}$.

9. Answer any *one* from the followings : 1×5=5

a) Solve $x\frac{dy}{dx} + y = y^2 \log x$

b) Find the general and singular solution of the differential equation $p^2 x(x-2) + p(2y - 2xy - x + 2) + y^2 + y = 0$.

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MATHEMATICS

Paper Code : UG/I/MATH/4/MJ-1T

Paper Name : Calculus, Geometry & Ordinery Differentiel Equations

Full Marks - 60 **Time - 3 hours**

The figures in the margin indicate full Marks. Candidates are required to give their own words as far as practicable. Illustrate the answers wherever necessary.

Unit-I (Differential Calculus)

[16 Marks]

1. Answer any *three* from the followings : 3×2=6

a) If $P_n = D^n(x^n \log x)$, prove that

$$P_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots.. + \frac{1}{n}\right)$$

b) Prove that in the Catenary, $y = a\cosh\left(\frac{x}{a}\right)$, the radius of curvature $\rho \propto y^2$.

c) Find the asymptotes of the curve $x^2y^2 - a^2(x^2 + y^2) - a^3(x + y) + a^4 = 0$ which are parallel to either axis.

d) Find the envelope of the family of lines $y = \alpha x + \frac{a}{\alpha}$, a is a variable parameter.

e) Find the points of inflexion if any of the curve $x = (\log y)^3$.

2. Answer any *one* from the followings : 1×10=10

a) i) If ρ_1, ρ_2 be the radii of curvature at the extremities of any chord of the cardiode $r = a(1 + \cos\theta)$, which passes through the pole, then prove that $\rho_1^2 + \rho_2^2 = \frac{16}{9}a^2$. 5

ii) Show that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $(x^2 + y^2)^2 = 16c^2xy$. 5

b) i) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ prove that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$ 5

ii) Find a, b, c such that $\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} \to 2$ as $x \to 0$. 5

Unit-II (Integral Calculus)

[14 Marks]

3. **Answer any *two* from the followings : 2×2=4**

a) Determine the length of one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

b) Find the reduction formula for $\int_0^{\frac{\pi}{4}} \tan^n x dx$, n being a positive integer.

c) Find the area of the region bounded by the curves $y = x^2$ and $x = y^2$.

4. **Answer any *two* from the followings : 2×5=10**

a) Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \cos^m x \cos nx dx$ where m, n are positive integers. Deduce that $\int_0^{\frac{\pi}{2}} \cos^m x \cos nx dx = \frac{\pi}{2^{m+1}}$, m being a positive integer.

b) Find the area of the surface generated by revolving about the y-axis that part of the asteriod $x = a\cos^3\theta$, $y = a\sin^3\theta$, that lies in the first quadrant.

c) If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$ then prove that $I_n = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\cdots\frac{1}{2}\frac{\pi}{2}$

where n is any even positive integer > 1.

Unit-III (Geometry)

[21 Marks]

5. **Answer any *three* from the followings : 3×2=6**

a) Find the equation of the sphere through the origin and meet the axes at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ respectively.

b) Prove that the length of the focal chord of a conic $\frac{1}{r} = 1 - e\cos\theta$ which is inclined to the axes at an angle α is $\frac{2l}{1 - e^2\cos^2\alpha}$.

c) Show that the equation $5x^2 + 3y^2 + z^2 - 6yz - 4zx - 2xy + 6x + 8y + 10z = 26$ represents a cone with vertex at the point $(1, 2, 3)$.

d) Find the value of m for which the plane $x - 2y - 2z + m = 0$ touches the ellipsoid $\frac{x^2}{144} + \frac{y^2}{36} + \frac{z^2}{9} = 1$.

e) If the axes are turned through an angle, the expression $ax + by$ becomes $a'y' + b'y'$ referred to new axes; show that $a^2 + b^2 = a'^2 + b'^2$.

6. **Answer any *one* from the followings : 1×5=5**

a) Reduce the equation $x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$ to its canonical form and find the nature of the conic.

b) If r_1 and r_2 be two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2\cos^2\theta}$, then show that $\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

7. **Answer any *one* from the followings : 1×5=5**

a) i) If PSP' and QSQ' be two perpendicular focal chord of a conic with focus S, then prove that $\frac{1}{SP.SP'} + \frac{1}{SQ.SQ'}$ is constant.

a) i) Find the equation of the plane passing through the point (1, 1, 2) and (2, 4, 3) and perpendicular to the plane $x - 3y + 7z + 5 = 0$.

ii) Reduce the equation $4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$ to canonial form and determine the nature of the conic.

b) i) Find the locus of the point of intersection of the perpenducular generators of the hyperbolic parabolid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$.

ii) Define enveloping cone and then find the equation of the enveloping cone.

Unit - IV(Differential Equation)

[9 marks]

8. Answer any *two* questions **2×2=4**

a) Find the differential equation of all circle $x^2 + y^2 = a^2$ where a being parameter.

b) Test if exact or not $y(1 + xy)dx + x(1 - xy)dy = 0$.

c) Is there any sigular solution of the differential equation $y = x\frac{dy}{dx} + a\frac{dx}{dy}$? Where a constant.

9. Answer any one question: **1×5=5**

a) Solve $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$

b) Find the general and singular solution of $\left(\frac{dy}{dx}\right)^3 + x\frac{dy}{dx} - y = 0$.

B.Sc. First Semester Examination(ESE) 2024

(CCFUP : NEP)

(3-Year UG- Programme)

MATHEMATICS

Paper Code : UG/I/MATH/3/MJ-A1T

Paper Name : Calculus, Geometry & Ordinery Differentiel Equations

Full Marks - 60 **Time - 3 hours**

The figures in the margin indicate full Marks. Candidates are required to give their own words as far as practicable. Illustrate the answers wherever necessary.

Unit-I(Differential Calculus)

[16 Marks]

1. Answer any *three* from the followings : **3×2=6**

a) Find the radius of curvature of $r = a(1 + \cos\theta)$, at any point θ

b) Find the vertical asymptote of the curve $y = xe^{\frac{1}{x}}$

c) Using L' Hospital's rule prove that $\lim_{x\to 0+}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}} = 1$

d) If $y = x^{n-1}\log x$, show that prove that $y_n = \frac{(n-1)!}{x}$

e) Show that the point of inflexion of the curve $y^2 = (x - a)^2(x - b)$ lies on the line $3x + a = 4b$.

2. Answer any *one* from the followings : **1×10=10**

a) i) Find the envelop of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are variable parameters, connected by the relation $a + b = c$, c being a non-zero constant. 5

ii) Find the radius of curvature at any point (r, θ) for the curve

$r^n = a^n \cot n^{\theta}$. 5

b) i) Find the asymptotes of the curve

$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$ 5

ii) Find if there is any point of inflexion on the curve

$y - 3 = 6(x - 2)^5$ 5

Unit-II (Internal Calculus)

[14 Marks]

3. **Answer any *two* from the followings : 2×2=4**

a) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinary every where except at the origin.

b) Obtain the reduction formula for $\int \sin^n x\,dx$

c) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$, then show that $I_{n+1} - I_{n-1} = \frac{1}{n}$

4. **Answer any *two* from the followings : 2×5=10**

a) Show that the volume and surface area of solid which is formed by revolving a cardiode $r = a(1-\cos\theta)$ about the initial line are $\frac{8}{3}\pi a^3$ and $\frac{32}{5}\pi a^2$.

b) If $I_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \sin nx\,dx$, where m, n are positive integers, show that $I_{m,n} = \frac{1}{m+n} + \frac{m}{m+n} I_{m-1,n-1}$. Hence deduce that $I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \ldots.. + \frac{2^m}{m}\right]$

c) Find the whole area included between the curve

$y^2 (a - x) = x^2 (a + x)$ and its asymptotem, $a > 0$.

Unit-III (Geometry)

[21 Marks]

5. **Answer any *three* from the followings : 3×2=6**

a) Transform to parallel axes through the point (–2, 3) the equation $2x^2 + 4xy + 3y^2 - 2x - 4y + 7 = 0$

b) Find the nature of the conic

$3x^2 + 2xy + 3y^2 - 16x + 20 = 0$

c) When the axes are turned through an angle, the expression $ax + by$ becomes $a'x' + b'y'$ referred to new axes, show that $a^2 + b^2 = a'^2 + b'^2$

d) Find k so that the equation

$kx^2 + 4xy + y^2 - 6x - 2y + 2 = 0$. may represent a point ellipse.

e) Deduce the equation of the right circular cylindar whose axis is $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

6. **Answer any *one* from the followings : 1×5=5**

a) Prove that the two conics

$\frac{l_1}{r} = 1 - e_1 \cos\theta$ *and* $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ will touch one another if $l_1^2\left(1 - e_2^2\right) + l_2^2\left(1 - e_1^2\right) = 2l_1l_2\left(1 - e_1e_2\cos\alpha\right)$

b) Find the sphere of the smallest radius that touches the straight lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z-6}{1}$ and $\frac{x+3}{7} = \frac{y+3}{-6} = \frac{z+3}{1}$

7. **Answer any *one* from the followings : 1×5=5**